Sensitivity analysis and optimisation of electromagnetic structures

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Abstract

A method is presented to analyse the sensitivity of the response of electromagnetic structures with respect to their design parameters and how to use this information for the optimisation of that response. It is shown that it is possible to determine the sensitivity in a closed-form analytical expression for different problem types. The sensitivity information is used for the optimisation of two distinct structures and it is shown that the response can be optimised within few iterations using a conjugate gradient optimisation algorithm. Moreover, it is shown that the optimisation process is quite robust, i.e., it is possible to find a global optimum for the problems under consideration.

I. INTRODUCTION

Gradient-based optimisation techniques have the advantage to be computationally efficient compared to stochastic optimisation techniques, especially in the case of large problems with many optimisation parameters. However, with gradient-based optimisation one is able to find only a local optimum and additional information about the derivative of the response function with respect to the optimisation parameters is needed. Therefore, the response function that has to be optimised should be chosen carefully, such that its derivatives can be calculated accurately and it should be verified that the optimisation routine does not come up with a local optimum that is not adequate.

A method is presented to analyse the sensitivity of the response of electromagnetic (EM) structures with respect to their design parameters and how to use this information for the optimisation of the response. The sensitivity, or gradient, to the design parameters is analysed with the direct differentiation method [1]. It is shown that it is possible to determine the sensitivity in a closed-form analytical expression for different design requirements. The sensitivity information is used for the optimisation of two distinct structures, i.e., a high-efficiency broadband microstrip antenna and a low-loss broadband vertical layer transition. Appropriate response functions for these optimisation problems are formulated and it is shown that the response can be optimised within few iterations using a conjugate gradient optimisation algorithm. Moreover, we show that the optimisation process is quite robust, i.e., it is possible to find a global optimum of the response function for the problems under consideration.

II. SENSITIVITY ANALYSIS

The electromagnetic problems that we consider are solved with the method of moments (MoM), in which the problem is represented as

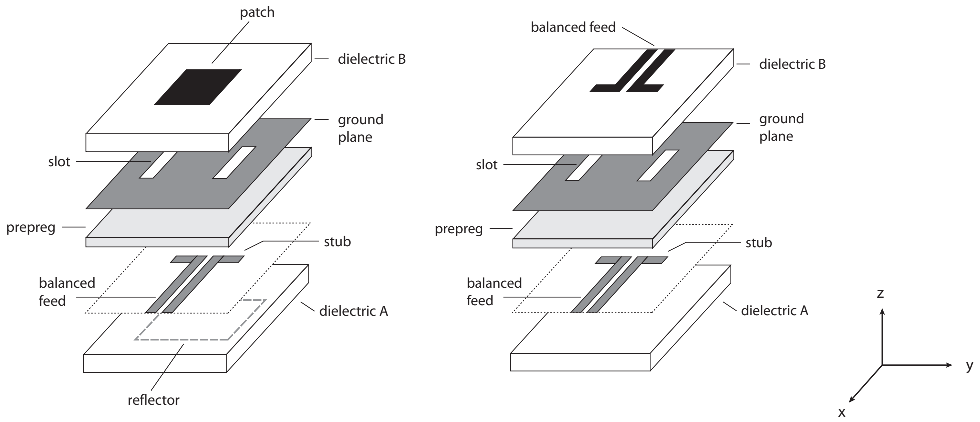
$$\mathbf{Z}(\mathbf{p})\mathbf{I} = \mathbf{V}(\mathbf{p}), \tag{1}$$

here $\mathbf{p}$ is a vector with design parameters, $\mathbf{I}$ is the state-variable vector, that represents complex currents or current densities and $\mathbf{V}$ is the excitation vector which, in general, represents the sources in the EM problem. The excitation vector may or may not depend on the parameters $\mathbf{p}$. The interaction matrix $\mathbf{Z}$ depends on the structure's geometry and materials and is in general dependent on the parameter vector $\mathbf{p}$.

The function $f(\mathbf{p})$ is introduced as the response function of the linear system. This function acts as a measure of the performance of the structure. The objective is to determine the gradient vector $\nabla_p f(\mathbf{p})$ of the response function with respect to the design parameters. For well-defined response functions, the gradient vector can be expressed in terms of $\nabla_p \mathbf{Z}$ and $\nabla_p \mathbf{V}$, as will be shown in Section III.

It is advantageous to have a closed-form analytical expression for the terms $\nabla_p \mathbf{Z}$, $\nabla_p \mathbf{V}$ such that the sensitivity can be calculated accurately. In this paper, the electromagnetic problems are related to planar structures within a stratified medium. The elements v_m, z_{mn} of the excitation vector and the interaction matrix are represented in the spectral domain representation [2]. For example, an element of the interaction matrix is written as

$$z_{mn}(\mathbf{p}) = \frac{1}{4\pi^2} \int\limits_{k_x} \int\limits_{k_y} \hat{\mathbf{J}}^t_m(\mathbf{p}, -k_x, -k_y, z) \cdot \left[\underline{\underline{\hat{\mathbf{G}}}}(\mathbf{p}, k_x, k_y, z, z_0) \cdot \hat{\mathbf{J}}^e_n(\mathbf{p}, k_x, k_y, z_0) \right] dk_x dk_y, \tag{2}$$



(a) Balanced-fed aperture-coupled patch antenna with reflector element [3]

(b) Vertical layer transition

Fig. 1. Geometries of the two considered EM problems

with $\mathbf{J}^t$ a test function, $\mathbf{J}^e$ an expansion function and $\underline{\underline{\mathbf{G}}}$ the dyadic Green's function used in the MoM analysis. From this expression it is easy to find an expression for $\nabla_p\overline{\overline{\mathbf{Z}}}$. It should be noted that in the spectral domain representation we are able to interchange the order of integration and differentiation and therefore we have a closed-form analytical expression for the gradient. In the spatial domain representation this is not always possible.

III. OPTIMISATION

In gradient-based optimisation techniques, a well-defined response function is very important as well as an adequate initial guess. In Sections III-A and III-B two examples are given, which show that these two requirements can often be fulfilled quite easily.

The two optimisation problems are both related to an antenna design for millimeter-wave communication [3]. The antenna layout is shown in Fig. 1a, whereas a vertical layer transition is shown in Fig. 1b. The first problem is how to optimise the bandwidth and the radiation efficiency of the antenna. The second problem is how to couple effectively from the balanced feed within the stack to a balanced feed at the upper layer of the stack with the use of two coupling slots.

A. Antenna optimisation

In this case we are interested to maximise the radiation efficiency $\eta(\omega, \mathbf{p})$ as well as to minimise the reflection coefficient $\Gamma(\omega, \mathbf{p})$ of the antenna over a certain bandwidth ($\omega_0 < \omega < \omega_1$). The accompanying response function is given as

$$f(\mathbf{p}) = \frac{1}{\omega_1 - \omega_0} \int_{\omega_0}^{\omega_1} (1 - |\Gamma(\omega, \mathbf{p})|^2)\eta(\omega, \mathbf{p}) d\omega. \tag{3}$$

The sensitivity of the integrand of the response function to the design parameters can be written as

$$\begin{aligned} \nabla_p\big[(1 - |\Gamma(\omega, \mathbf{p})|^2)\eta(\omega, \mathbf{p})\big] = &- 2(\Re\{\Gamma(\omega, \mathbf{p})\}\Re\{\nabla_p\Gamma(\omega, \mathbf{p})\} \\ &+ \Im\{\Gamma(\omega, \mathbf{p})\}\Im\{\nabla_p\Gamma(\omega, \mathbf{p})\})\eta(\omega, \mathbf{p}) + (1 - |\Gamma(\omega, \mathbf{p})|^2)\nabla_p\eta(\omega, \mathbf{p}), \end{aligned} \tag{4}$$

here, $\nabla_p\Gamma$ and $\nabla_p\eta$ can be expressed in terms of $\nabla_p\mathbf{V}$ and $\nabla_p\mathbf{Z}$.

The initial design that has been realised and measured [3] operates in the band from 54 to 60 GHz and has a radiation efficiency which is larger than 80%. New applications require the antenna to operate in the frequency band from 57 to 64 GHz. Obviously, the radiation efficiency should be as large as possible in this band, while the reflection coefficient should be as small as possible. Therefore, the length of the patch, the length of the slots, the spacing of the slots and the length of the stubs are adjusted such that the response function is optimised.

The response function was optimised through a conjugate gradient optimisation algorithm. The convergence of the response function is shown in Fig. 2. The reflection coefficient and the accompanying radiation efficiency of the resulting antenna design are shown in Figure 3. It is observed that this algorithm is able to find an optimum in just a few number of iterations. It appears that this antenna geometry is able to achieve the required bandwidth with a radiation efficiency that is larger than 79% throughout the band of operation.

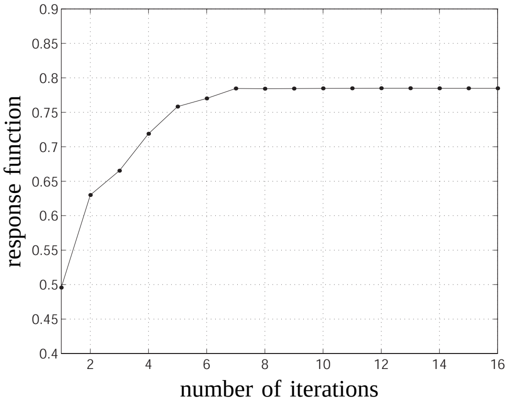


Fig. 2. Convergence of response function

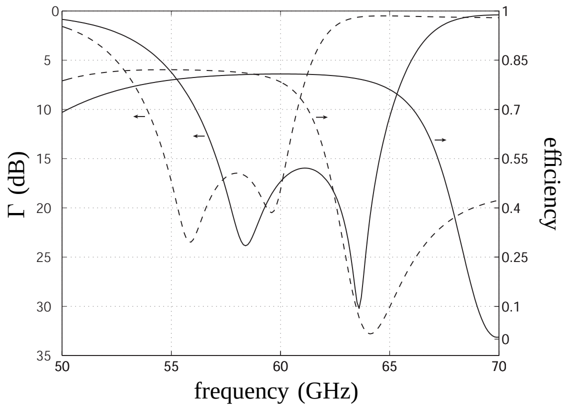


Fig. 3. Reflection coefficient and efficiency antenna. Initial design (dashed), optimised (solid).

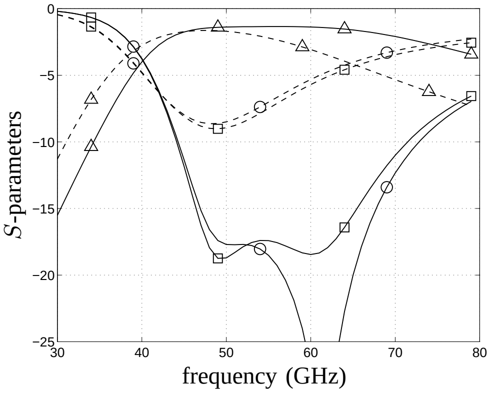


Fig. 4. S-parameters vertical layer transition. Initial guess (dashed), optimised (solid). $S_{11}(\bigcirc)$, $S_{12}(\triangle)$, $S_{22}(\square)$

B. Vertical layer transition optimisation

The vertical layer transition has been optimised for the transmission coefficient $S_{12} = S_{21}$ in the frequency band from 50 to 70 GHz. The accompanying response function is given as

$$f(\mathbf{p}) = \frac{1}{\omega_1 - \omega_0} \int_{\omega_0}^{\omega_1} |S_{12}|^2 d\omega, \tag{5}$$

here, the design parameters are the length of the stubs in the lower layer, the length of the slots and the length of the stubs in the upper layer. The performance of the initial design and the optimised design is shown in Fig. 4. The optimisation routine was converged within 10 iterations. A poor initial guess was chosen to show the robustness of the optimisation process. Moreover, different initial guesses have been tried to verify that the optimisation routine finds a unique optimum. The optimised design shows a transmission loss below 2 dB in the frequency range from 44 to 69 GHz. Resultantly, also the reflection coefficients improve significantly.

IV. CONCLUSIONS

A method has been presented that determines the sensitivity of the response of EM structures with respect to their design parameters. This information has been used for the optimisation of the response. It has been shown that a gradient-based optimisation technique is capable to optimise a variety of structures within just a few iterations. Moreover it has been shown that adequate response functions can be formulated such that the optimised structure does not dependent on the initial guess of the optimisation routine.

REFERENCES

[1] S. Amari, P. Harscher, R. Vahldieck, and J. Bornemann, “Novel analytic gradient evaluation techniques for optimization of microwave structures,” *IEEE MTT-S Int. Microwave Symp. Digest*, vol. 1, pp. 31–34, June 1999.

[2] D. Pozar, “Input impedance and mutual coupling of rectangular microstrip antennas,” *IEEE trans. on Antennas and Propagation*, vol. 30, pp. 1191–1196, November 1982.

[3] J. Akkermans, M. van Beurden, and M. Herben, “Design of a millimeter-wave balanced-fed aperture-coupled patch antenna,” in *proc. EuCAP 2006, ESA SP626*, (Nice, France), pp. 1–6, November 2006.