Combined Multiple Transmit Antennas and Multilevel Modulation Techniques

Umar H. Rizvi, Gerard J. M. Janssen
IRCTR/CWPC, WMC Group,
Faculty of EEMCS,
Delft University of Technology
Delft, The Netherlands.
{u.h.rizvi,g.janssen}@ewi.tudelft.nl

S. Ben Slimane
Radio Communication Systems,
Dept. of Communication Systems,
The Royal Institute of Technology (KTH),
Stockholm, Sweden.
slimane@radio.kth.se

Abstract

The use of a unique bit-to-symbol mapping for each diversity branch (often referred to as constellation rearrangement) is known to provide good performance for higher level linear modulation techniques when used in conjunction with orthogonal transmit diversity (OTD) and automatic repeat request (ARQ) schemes. This paper investigates the performance of constellation rearrangement scheme when used in conjunction with multiple transmit antennas and space time block codes. A modified space-time block coding structure is presented that employs a different constellation mapping on each diversity branch. It is seen that, the use of different constellation mappings in the space time block coded systems provide a better performance as compared to the OTD systems while maintaining the same spectral efficiency.

1 Introduction

The capacity of band-limited, fading wireless communication channels can be drastically increased by employing multiple antennas (or space diversity) at the transmitter and the receiver [1]. The added dimension *space* to the existing dimension *time* opened the possibility of using efficient two-dimensional channel codes known as *space-time codes*. A simple space-time block code (STBC) for 2 transmit antennas was first introduced by Alamouti [2] which was later generalized by Tarokh et al. [3] for a larger number of transmit antennas . Another form of such codes known as space-time trellis codes (STTCs) constructed by jointly designing channel coding, modulation and transmit diversity was proposed by Tarkoh et. al. [4]. STTCs exhibit a superior performance as compared to STBC, but at the expense of increased complexity. Recently Slimane [5, 6] showed that a conventional orthogonal transmit diversity system could be improved by using different constellation mappings on each of the orthogonal transmit diversity branches. Such a scheme is also referred to as permutation coding [7].

This paper considers the use of different constellation mappings with multiple transmit antennas instead of orthogonal transmit diversity [5]. The main idea is to use optimized constellations for each space-time coded symbol by taking advantage of the conventional space-time block coding orthogonality [2, 3]. This scheme provides some coding gain in addition to the diversity and Euclidean distance gain of the orthogonal transmit diversity with constellation rearrangement scheme. It still however imposes a bandwidth inefficiency which is 1/2 for two transmit antennas. It is also seen that, contrary to the OTD scheme where the bandwidth efficiency is inversely proportional to the number of orthogonal transmit branches, the bandwidth efficiency in the STBC based constellation rearrangement (STBC-CR) scheme is dependent on the structure of STBC and not on the number of transmit branches (or antennas).

Section 2 provides an overview of conventional and constellation rearrangement based OTD systems. The principle of conventional space-time block codes (STBCs) along with their extension to incorporate constellation rearrangement is explained in Section 3. In Section 4, we present a numerical upper bound that can be used for the performance evaluation of the STBC-CR scheme. The performance curves are presented in Section 5, followed by conclusions in Section 6.

2 Orthogonal Transmit Diversity and Constellation Rearrangement

The input/output relationship of a two branch OTD scheme [5, 6] is given by

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_m^1 & 0 \\ 0 & s_m^2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{1}$$

where r_1, r_2 represent the received signals, s_m^1, s_m^2 are the transmitted symbols chosen from the set of M possible constellation points (i.e. for 4PAM, $M = 4$), h_1, h_2 represent independent and identically distributed (i.i.d) Rayleigh random variables with $E\{h_1^2\} = E\{h_2^2\} = 1$ and n_1, n_2 represent the i.i.d complex Gaussian random variables with zero mean and variance $N_0/2$ per dimension. The increase in signal spread of the constellation points can be explained by taking 4PAM as an example. Figure 1 gives the signal constellation for both the conventional and the rearranged case. In case of conventional OTD both constellation sets s_m^1, s_m^2 use the same bit to symbol mapping as illustrated in Figure 1. It has been shown in [5, 6, 8] that an appropriate choice of bit to symbol mapping on each diversity branch results in an increased Euclidean distance and thus provides better performance (as is evident from Figure 1). The bandwidth efficiency of an OTD scheme is inversely proportional to the number of diversity branches, therefore 4PAM and 2 branch diversity system has an effective transmission rate of 1 bit/s/Hz. Intuitively, it is interesting to note here that the symbol that is transmitted at a higher energy on branch one, has a lower associated energy level in branch two. Thus the mapping rearrangement has the effect of equalizing the average transmitted energy per symbol. It is important to note here that the rearranged constellations are optimized in terms of symbol error rate performance rather than bit error rate performance.

Assuming that the receiver has access to perfect channel information, the maximum likelihood (ML) detector chooses for the symbol $s_{\hat{m}}$, that minimizes the metric

$$\mathcal{C}(\hat{m}) = \left|r_1 - h_1 s_{\hat{m}}^1\right|^2 + \left|r_2 - h_2 s_{\hat{m}}^2\right|^2. \tag{2}$$

The constellation mapping on each branch can be chosen by inspection (for small constellation sets) as was done in Figure 1 or by performing a computer search (for larger constellations). The objective is to choose the mapping on each branch such that the squared Euclidean distance (SED) between any two signal constellation points $d_m = |s_m^1 - s_m^2|^2$ is maximized.

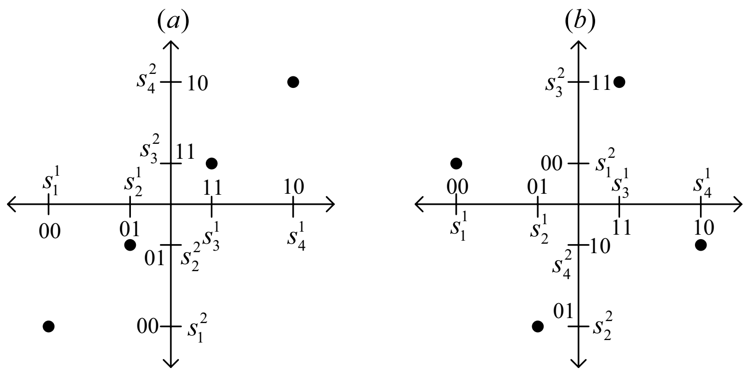

Figure 1: Conventional (a) and rearranged (b) signal constellation points for 2 branch OTD using 4PAM constellations

3 Space-Time Codes and Constellation Rearrangement

The space-time block coding (STBC) scheme [2] employing two transmit antennas is given by

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_m & s_k \\ s_k^* & -s_m^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{3}$$

where in the above equation the rows represent the time slots and columns the antennas i.e. at time slot t_1 symbols s_m and s_k are transmitted from antenna 1 and 2 respectively and at time slot t_2 symbols s_m and s_k are transmitted. The received symbols on each time slot t_1 and t_2 are denoted by r_1 and r_2 respectively. The channel fading coefficients h_1, h_2 are taken to be Rayleigh distributed and assumed to be stay constant for two successive time slots. In the case of 2 transmit antennas 2 symbols are transmitted in t2 time slots therefore the Alamouti STBC scheme has rate 1. The ML detector chooses for the symbols $s_{\hat{m}}, s_{\hat{k}}$, that minimize the metric

$$\mathcal{C}(\hat{m},\hat{k}) = \left|r_1 - (h_1 s_{\hat{m}} + h_2 s_{\hat{k}})\right|^2 + \left|r_2 - \left(h_1 s_{\hat{k}}^* - h_2 s_{\hat{m}}^*\right)\right|^2. \tag{4}$$

The rate 3/4 STBC scheme [4] using 4 transmit antennas is given as

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} s_m & s_k & \frac{s_l}{\sqrt{2}} & \frac{s_l}{\sqrt{2}} \\ -s_k^* & s_m^* & \frac{s_l}{\sqrt{2}} & -\frac{s_l}{\sqrt{2}} \\ \frac{s_l^*}{\sqrt{2}} & \frac{s_l^*}{\sqrt{2}} & \frac{-s_m - s_m^* + s_k - s_k^*}{\sqrt{2}} & \frac{-s_k - s_k^* + s_m - s_m^*}{\sqrt{2}} \\ \frac{s_l^*}{\sqrt{2}} & -\frac{s_l^*}{\sqrt{2}} & \frac{s_k + s_k^* + s_m - s_m^*}{\sqrt{2}} & \frac{-s_m - s_m^* + s_k - s_k^*}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}. \tag{5}$$

The ML detector chooses for the symbols $s_{\hat{m}}, s_{\hat{k}}, s_{\hat{l}}$, that minimize the metric

$$\mathcal{C}\left(\hat{m},\hat{k},\hat{l}\right) = \left|r_1 - \left(h_1 s_{\hat{m}} + h_2 s_{\hat{k}} + h_3\frac{s_{\hat{l}}}{\sqrt{2}} + h_4\frac{s_{\hat{l}}}{\sqrt{2}}\right)\right|^2 + \left|r_2 - \left(-h_1 s^*_{\hat{k}} + h_2 s^*_{\hat{m}} + h_3\frac{s_{\hat{l}}}{\sqrt{2}} - h_4\frac{s_{\hat{l}}}{\sqrt{2}}\right)\right|^2 + \left|r_3 - \left(h_1\frac{s^*_{\hat{l}}}{\sqrt{2}} + h_2\frac{s^*_{\hat{l}}}{\sqrt{2}} + h_3\left(\frac{-s_{\hat{m}} - s^*_{\hat{m}} + s_{\hat{k}} - s^*_{\hat{k}}}{\sqrt{2}}\right) + h_4\left(\frac{-s_{\hat{k}} - s^*_{\hat{k}} + s_{\hat{m}} - s^*_{\hat{m}}}{\sqrt{2}}\right)\right)\right|^2 + \left|r_4 - \left(h_1\frac{s^*_{\hat{l}}}{\sqrt{2}} - h_2\frac{s^*_{\hat{l}}}{\sqrt{2}} + h_3\left(\frac{s_{\hat{k}} + s^*_{\hat{k}} + s_{\hat{m}} - s^*_{\hat{m}}}{\sqrt{2}}\right) + h_4\left(\frac{-s_{\hat{m}} - s^*_{\hat{m}} + s_{\hat{k}} - s^*_{\hat{k}}}{\sqrt{2}}\right)\right)\right|^2. \quad (6)$$

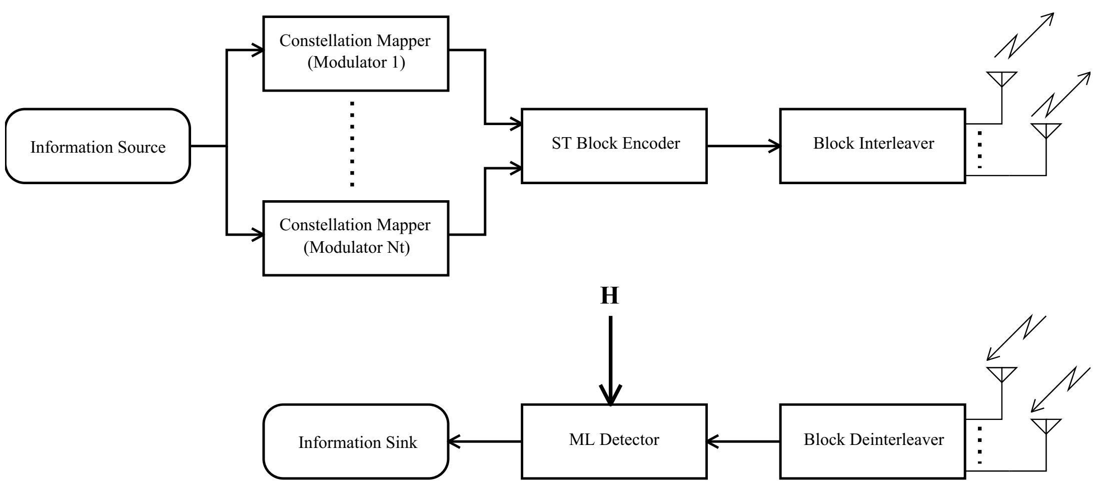

Figure 2: Combined constellation rerrangement and STBC system

The constellation rearrangement (CR) scheme explained in Section XX holds good only when different transmit branches are separated in time and/or frequency. The use of CR scheme in conjunction with multiple transmit antennas employing STBC is outlined next. Figure 2 gives the block diagram of a STBC-CR system. The source information bits are simultaneously passed to each of the constellation mapper(s) that use different mapping for each branch. The modulated symbols are then passed to the conventional space-time block encoder. The input/output relationship for the STBC scheme combined with constellation rearrangement is given by

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s^1_m & s^2_m \\ s^{2*}_m & -s^{1*}_m \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}. \quad (7)$$

The ML detector chooses for the symbol $s_{\hat{m}}$, that minimizes the metric

$$\mathcal{C}(\hat{m}) = \left|r_1 - \left(h_1 s^1_{\hat{m}} + h_2 s^2_{\hat{m}}\right)\right|^2 + \left|r_2 - \left(h_1 s^{2*}_{\hat{m}} - h_2 s^{1*}_{\hat{m}}\right)\right|^2. \quad (8)$$

As in the case of 2 branch OTD scheme, the diversity coupled STBC scheme also has an effective transmission rate that is one half as compared to the conventional STBC scheme. The 2 branch OTD scheme and the STBC with constellation rearrangement however, have the same spectral efficiency.

The received signal for STBC scheme combined with CR employing 4 transmit antennas is given as

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} s_m^1 & s_m^2 & \frac{s_m^3}{\sqrt{2}} & \frac{s_m^3}{\sqrt{2}} \\ -s_m^{2*} & s_m^{1*} & \frac{s_m^3}{\sqrt{2}} & -\frac{s_m^3}{\sqrt{2}} \\ \frac{s_m^{3*}}{\sqrt{2}} & \frac{s_m^{3*}}{\sqrt{2}} & \frac{-s_m^1-s_m^{1*}+s_m^2-s_m^{2*}}{\sqrt{2}} & \frac{-s_m^2-s_m^{2*}+s_m^1-s_m^{1*}}{\sqrt{2}} \\ \frac{s_m^{3*}}{\sqrt{2}} & -\frac{s_m^{3*}}{\sqrt{2}} & \frac{s_m^2+s_m^{2*}+s_m^1-s_m^{1*}}{\sqrt{2}} & \frac{-s_m^1-s_m^{1*}+s_m^2-s_m^{2*}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}. \quad (9)$$

The estimated symbol $s_{\hat{m}}$ is chosen based on the following ML decision rule

$$\begin{aligned} \mathcal{C}(\hat{m}) = & \left| r_1 - \left(h_1 s_{\hat{m}}^1 + h_2 s_{\hat{m}}^2 + h_3 \frac{s_{\hat{m}}^3}{\sqrt{2}} + h_4 \frac{s_{\hat{m}}^3}{\sqrt{2}} \right) \right|^2 + \\ & \left| r_2 - \left(-h_1 s_{\hat{m}}^{2*} + h_2 s_{\hat{m}}^{1*} + h_3 \frac{s_{\hat{m}}^3}{\sqrt{2}} - h_4 \frac{s_{\hat{m}}^3}{\sqrt{2}} \right) \right|^2 + \\ & \left| r_3 - \left(h_1 \frac{s_{\hat{m}}^{3*}}{\sqrt{2}} + h_2 \frac{s_{\hat{m}}^{3*}}{\sqrt{2}} + h_3 \left(\frac{-s_{\hat{m}^1} - s_{\hat{m}}^{1*} + s_{\hat{m}}^2 - s_{\hat{m}}^{2*}}{\sqrt{2}} \right) + h_4 \left(\frac{-s_{\hat{m}}^2 - s_{\hat{m}}^{2*} + s_{\hat{m}}^1 - s_{\hat{m}}^{1*}}{\sqrt{2}} \right) \right) \right|^2 + \\ & \left| r_4 - \left(h_1 \frac{s_{\hat{m}}^{3*}}{\sqrt{2}} - h_2 \frac{s_{\hat{m}}^{3*}}{\sqrt{2}} + h_3 \left(\frac{s_{\hat{m}}^2 + s_{\hat{m}}^{2*} + s_{\hat{m}}^1 - s_{\hat{m}}^{1*}}{\sqrt{2}} \right) + h_4 \left(\frac{-s_{\hat{m}}^1 - s_{\hat{m}}^{1*} + s_{\hat{m}}^2 - s_{\hat{m}}^{2*}}{\sqrt{2}} \right) \right) \right|^2 . \end{aligned} \quad (10)$$

In the STBC-CR scheme a different symbol mapping is used based on the STBC block encoding structure unlike the OTD-CR scheme where a different mapping is used for each diversity branch. The use of STBC-CR scheme is therefore dependent on the STBC structure. In case of OTD scheme with 4 orthogonal transmit branches 4 different sets of constellation mappings will be required as opposed to 3 in case of STBC-CR scheme of 4 transmit antennas.

4 Performance Analysis

This section presents a numerical upper bound for the symbol error probability of the STBC-CR scheme. For an AWGN channel, the pairwise error probability is given by [5]

$$P(m \to \hat{m}) = Q\left(\sqrt{\frac{D^2(s_m, s_{\hat{m}})}{2N_0}} \right), \quad \text{where} \quad Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} \exp^{-t^2/2} dt, \quad (11)$$

and

$$D^2(s_m, s_{\hat{m}}) = \frac{1}{N_t} \sum_{i=1}^{N_t} \left| s_m^i - s_{\hat{m}}^i \right|^2, \quad (12)$$

is the squared Euclidean distance between the transmitted sequence s_m^i and the detected sequence $s_{\hat{m}}^i$ and N_t denotes the number of transmit branches. Averaging over all possible candidate symbols, an upper bound on the symbol error probability is given by [9, p. 172]

$$P_s \leq \frac{1}{M} \sum_{m=1}^{M} \sum_{\hat{m} \neq m} Q\left(\sqrt{\frac{D^2(s_m, s_{\hat{m}})}{2N_0}} \right) \quad (13)$$

where M is the total number of symbols in the modulation set.

For the STBC-CR of (7), over a Rayleigh Fading channel, with uncorrelated branches, an upper bound on the symbol error probability can be derived in a similar manner. For given fading channel observations h_1, h_2, an upper bound on the symbol error probability of combined constellation rearrangement with STBC can be written as

$$P_s \leq \frac{1}{M} \sum_{m=1}^{M} \sum_{\hat{m} \neq m} Q\left(\sqrt{\frac{D_h^2(s_m, s_{\hat{m}})}{2N_0}}\right) \tag{14}$$

where

$$D_h^2(s_m, s_{\hat{m}}) = \left(|h_1|^2 + |h_2|^2\right)\left(\left|s_m^1 - s_{\hat{m}}^1\right|^2 + \left|s_m^2 - s_{\hat{m}}^2\right|^2\right) \tag{15}$$

In case of a 2 branch OTD-CR system [5, 6], the distance metric $D_h^2(s_m, s_{\hat{m}})$ is given as

$$D_h^2(s_m, s_{\hat{m}}) = |h_1|^2 \left|s_m^1 - s_{\hat{m}}^1\right|^2 + |h_2|^2 \left|s_m^2 - s_{\hat{m}}^2\right|^2 \tag{16}$$

Comparing expressions (15) for the STBC-CR scheme and (16) for the OTD-CR scheme, it can be seen that the gain in the STBC-CR scheme is due to an increase in the Euclidean distance. We can see that the diversity gain and the Euclidean distance gain are separated and they add in case of STBC-CR scheme whereas this is not the case for OTD-CR scheme where there is a mixture between the two.

5 Simulation Results

We illustrate the performance of the modified STBC-CR using 16QAM and 64QAM modulation schemes and different numbers of transmit antennas. The wireless multipath channel is assumed to be slowly Rayleigh fading and uncorrelated between different branches.

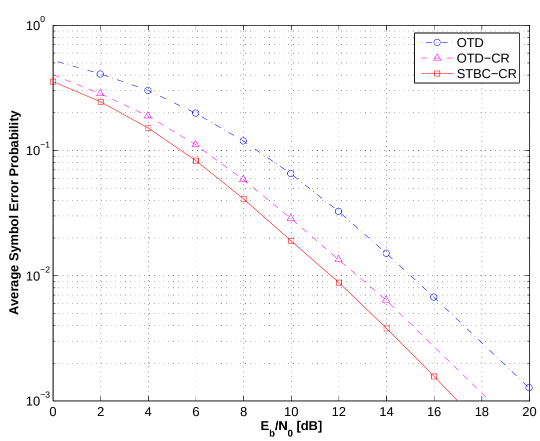

Figure 3: Average symbol error probability of 16QAM over Rayleigh fading channels ($N_t = 2$)

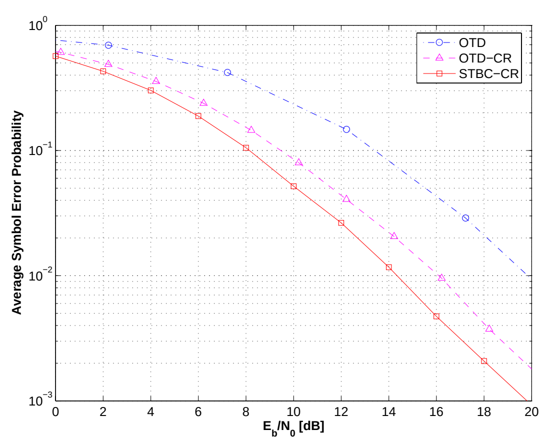

Figure 4: Average symbol error probability of 64QAM over Rayleigh fading channel ($N_t = 2$)

Simulation results for the 16QAM system employing two transmit antennas and one receive antenna are depicted in Figure 3. The obtained results show an improvement of 1.2 dB as compared to orthogonal transmit diversity with constellation rearrangement at a SEP of 10^{-3}.

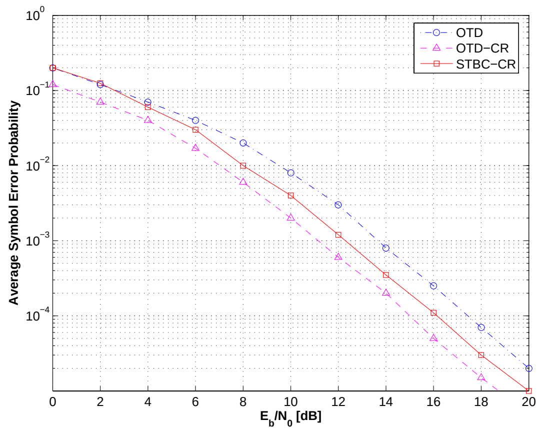

Figure 5: Average symbol error probability of 16QAM over Rayleigh fading channels ($N_t = 4$)

Figure 4 gives the simulation results for the average symbol error probability of a employing 64QAM modulation STBC-CR scheme employing 2 transmit antennas. At a SEP of 10^{-2}, the STBC-CR is seen to outperform the OTD-CR by 1.8 dB.

Figure 5 gives the performance curves for STBC-CR with 4 transmit antennas and 16QAM modulation scheme. In this case the new scheme actually performs 1 dB worse with reference to the OTD-CR. This can be explained by the fact that, the constellation optimization strategy provides better product distance as we increase the number of branches and/or modulation level [5, 6]. If we now consider the structure of the Tarokh rate 3/4 space-time code, it transmits 3 symbols on 4 time slots which therefore requires 3 branch optimized constellations as opposed to the 4 branch optimized constellations, used in the OTDC-CR scheme. Therefore a better signal spread is obtained in the OTD-CR scheme as compared to the STBC-CR scheme at the same spectral efficiency when 4 transmit branches are used. On a more intuitive note the increased combined diversity, coding and Euclidean distance gain of the modified scheme cannot outperform the diversity and Euclidean distance gain of the OTD-CR scheme. To the best of our knowledge there exists no orthogonal rate 1 STBC scheme, that utilizes 4 transmit antennas and complex constellations.

6 Conclusions

This paper presented the use of space-time block coding scheme coupled with constellation rearrangement for multi-input multi-output wireless communication systems. The use of different constellation maps on each branch in STBC-CR scheme is dependent on the underlying STBC structure. The obtained simulation results show that space time block codes with constellation rearrangement provide a better performance as compared to orthogonal transmit diversity based constellation rearrangement scheme when used with 2 transmit antennas. In case of 4 transmit antennas the OTD-CR scheme is seen to outperform the STBC-CR scheme as it employs 4 different constellation mappings as opposed to 3 for the STBC-CR scheme. The results presented in this paper only considered space-time block codes. Combined constellation rearrangement and space-time trellis codes (STTC) and performance evaluation under more realistic correlated fading channels is a natural extension to this work.

Acknowledgment This work was supported by IOP Gen Com under SiGi Spot project IGC.0503.

References

[1] G. Foschini and M. Gans, "On the limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Communications,* vol. 6, pp. 311-335, March 1998.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communication," *IEEE Journal of Select. Areas Comm.*, vol. 16, pp. 1451-1458, Oct. 1998.

[3] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, July 1999.

[4] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Information Theory,* vol. 44, no. 2, March 1998.

[5] S. B. Slimane, "Combined transmit diversity and multi-level modulation techniques," *Proc. IEEE SETIT,* March 2005.

[6] S. B. Slimane, "Combined transmit diversity and multi-level modulation techniques," *Wireless Personal Communications, Kluwer Academic Publishers,* October 2006.

[7] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication,* Cambridge University Press, 2005.

[8] H. Samra, Z. Ding and P. M. Hahn, "Optimal symbol mapping diversity for multiple packet transmissions,"Proc. IEEE ICASSP, vol. 4, pp. 181–184, April 2003.

[9] J. Zander, S. B. Slimane and L. Ahlin, *Principles of Wireless Communications*, Studentlitteratur, 3rd ed., Lund, Sweden, 2004.