ROTATION OPTIMIZATION FOR MPSK/MQAM SIGNAL CONSTELLATIONS OVER RAYLEIGH FADING CHANNELS

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ABSTRACT

The performance of uncoded phase-shift-keying (PSK) and quadrature amplitude modulation (QAM) schemes over fading channels can be improved by using coded modulation techniques. Improvement is due to the coding gain coupled with interleaving and depends on the complexity of the code. Recently, it was shown that constellation rotation coupled with interleaving can be used to improve the performance of QPSK modulation over block-fading single-input-single-output (SISO) wireless communication channels. This paper considers the use of such a scheme with higher order constellation sets such as 8PSK and 16QAM. A framework is then presented for the calculation of the optimum rotation angle for MPSK/MQAM schemes. A simple cost function based on the union bound of the symbol error probability (SEP) is defined. The optimum rotation angle is then found by minimizing the cost function using the gradient search algorithm. The obtained simulation results show considerable improvement over the conventional unrotated system.

1. INTRODUCTION

The fourth-generation (4G) wireless communication systems are expected to provide a number of high data rate services in outdoor, indoor and pico-cellular applications [1]. The research aims at developing efficient techniques that can support high data rates through band-limited wireless communication channels. Fading causes significant performance degradation in wireless digital communication systems. For block fading channels, improved performance can be obtained by the use of coded modulation techniques coupled with interleaving [2], [3], [4]. It was argued that minimum squared Euclidean distance is a secondary error event criteria over fading channels. Therefore an optimum scheme for the additive white Gaussian noise (AWGN) channel may not be the best possible solution for fading channels.

It was shown in [5] that for a block-fading wireless communication link, diversity can be introduced into the system by separately interleaving the inphase and quadrature components of a QPSK scheme and performing symbol-by-symbol detection. It was argued that the performance of such a scheme depends on the constellation rotation angle and has no effect when used in conjunction with an AWGN channel. The impact of rotation on the performance of fading channels was also outlined in [6].

In this paper we present a method of finding the optimum rotation angle for such a transmission scheme. This method can be used to find the optimum rotations for any complex linear multilevel modulation scheme (MPSK/MQAM). It is shown that the rotated system provides performance gains over the conventional and unrotated scheme and also extends the suboptimum QPSK scheme of [5], to higher multilevel linear modulation formats.

Section 2 outlines the system model which is used to investigate the performance of constellation rotation. The framework for computing the optimum rotation angle for fading channels is presented in Section 3. In Section 4, we present performance curves for optimal rotations. Conclusions are drawn in Section 5.

2. SYSTEM MODEL

Any conventional MQAM/MPSK (complex) modulation scheme can be seen as two (real) M-ary pulse amplitude modulations (MPAMs) in parallel– one on the inphase (I channel) and one on the quadrature phase (Q channel). By generalizing the notation given in [5], a conventional MPSK/MQAM scheme at a carrier frequency f_c can be represented by

$$s(t) = \sum_{i=-\infty}^{+\infty} a_i p(t - iT_s)\cos(2\pi f_c t) + \sum_{i=-\infty}^{+\infty} b_i p(t - iT_s)\sin(2\pi f_c t) \quad (1)$$

where

$$p(t) = \begin{cases} 1, & 0 \le t \le T_s, \\ 0, & \text{otherwise} \end{cases}$$

The parameters a_i, b_i are modulation specific constants that are assumed to be equiprobable and are defined as given in Table I. Note that the coefficients were chosen such that the average transmitted energy is constrained to unity. It was shown in [5] that by rotating the signal constellation and separately interleaving the I and Q components, an improved

TABLE I

a_i, b_i VALUES FOR VARIOUS MODULATION SCHEMES

Modulation	a_i	b_i
QPSK	$\left\{\pm\sqrt{1/2}\right\}$	$\left\{\pm\sqrt{1/2}\right\}$
8PSK	$\left\{\pm\sqrt{1/2},\ \pm 1,\ 0\right\}$	$\left\{\pm\sqrt{1/2},\ 0,\ \pm 1\right\}$
16QAM	$\left\{\pm\sqrt{1/10},\ \pm\sqrt{1/10},\ \pm\sqrt{9/10},\ \pm\sqrt{9/10}\right\}$	$\left\{\pm\sqrt{1/10},\ \pm\sqrt{9/10},\ \pm\sqrt{1/10},\ \pm\sqrt{9/10}\right\}$

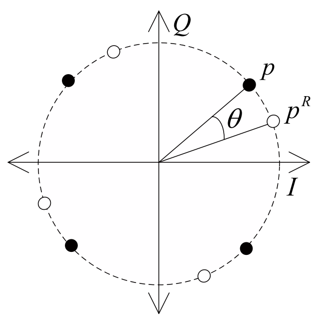

Fig. 1. Signal constellation rotation

performance can be obtained for a QPSK system without affecting its bandwidth efficiency.

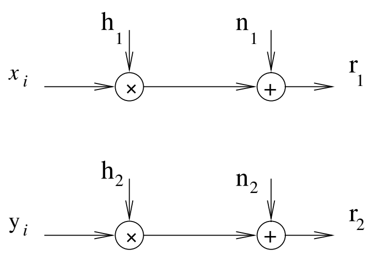

Fig. 2. Equivalent baseband model

Consider the signal constellation given in Figure 1. The points p^R and p are related by the following simple transformation

$$[p_I^R \quad p_Q^R] = [p_I \quad p_Q] \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{2}$$

Thus the I and Q components after a *clockwise* rotation of θ can be written as

$$\begin{aligned} x_i &= a_i \cos\theta + b_i \sin\theta \\ y_i &= -a_i \sin\theta + b_i \cos\theta \end{aligned} \tag{3}$$

The transmission scheme for a rotated system is given by

$$\begin{aligned} s(t) = &\sum_{i=-\infty}^{+\infty} x_i p(t - iT_s)\cos(2\pi f_c t) \\ &+ \sum_{i=-\infty}^{+\infty} y_{i-k} p(t - iT_s)\sin(2\pi f_c t) \end{aligned} \tag{4}$$

where k is an integer representing the time delay in number of symbols introduced by interleaving between I and Q components. Figures 4 and 5 give the block diagram for such a system [5]. The signal interleavers are chosen such that after deinterleaving the two components will be independent. Separate interleaving of I and Q components is analogous to transmitting the I component (x_i) during one fade interval and the Q component (y_i) during the next fade interval. The joint detection is performed at the receiver using two separately deinterleaved components. This sort of interleaving adds diversity in the system as x_i and y_i experience independent fading.

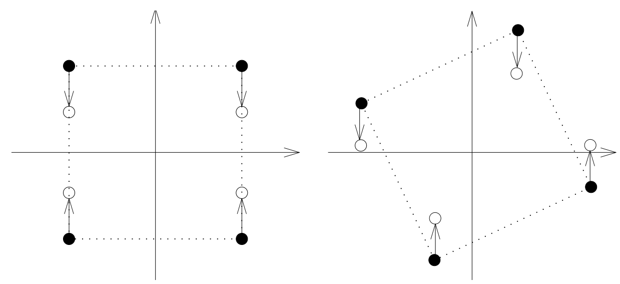

Fig. 3. Constellation rotation and fading

The fully interleaved system depicted in the Figures 4 and 5 can be modeled (in baseband) as two parallel Rayleigh fading channels as shown in Figure 2. For a fading wireless channel the input/output relation per channel use can be modeled by the following complex baseband relationship

$$r_1 = h_1 x_i + n_1 \tag{5}$$

$$r_2 = h_2 y_i + n_2 \tag{6}$$

Under the assumption of flat Rayleigh fading, the coefficients h_i can be modeled by a magnitude and phase distortion i.e. $h_i = |\alpha_i| e^{j\theta_i}$, where for a rich scattering environment h_i follows a Rayleigh distribution and θ_i is uniformly distributed on the interval $[0, 2\pi]$. The complex noise components n_i in (5) are independent and identically distributed (i.i.d) Gaussian random variables with zero mean and variance $N_0/2$ (i.e. $n_i \sim \mathcal{N}(0, N_0/2)$, per complex channel). Assuming perfect channel state information is available at the receiver, joint symbol by symbol detection can be performed at the receiver using the ML decision metric

$$\mathcal{C}(i, \hat{i}) = |r_1 - h_1 \hat{x_i}|^2 + |r_2 - h_2 \hat{y_i}|^2 \tag{7}$$

The detector thus chooses in favor of the symbol $\hat{s_i} = (\hat{x_i}, \hat{y_i})$, that minimizes the above metric.

The reason why constellation rotation works can intuitively be explained by Figure 3. Assuming that one of the channels

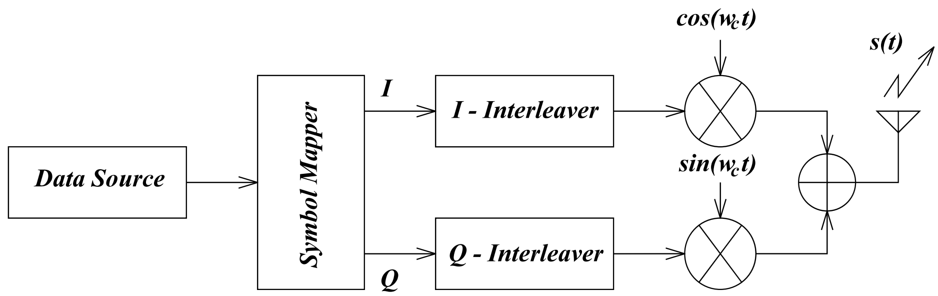

Fig. 4. Modulator block diagram

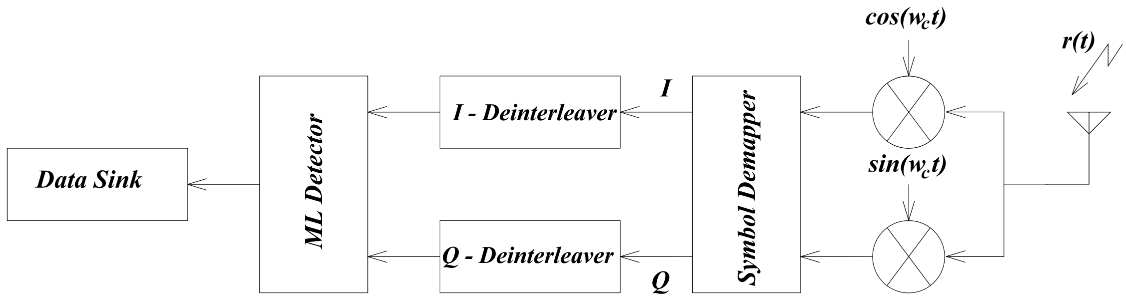

Fig. 5. Demodulator block diagram

experiences a very bad fade, the unrotated and rotated constellations shrink in only one of the branches. As can be clearly seen from Figure 3, the constellation points of the rotated scheme are separated by a greater Euclidean distance and thus provide good performance. No fades are experienced in the Gaussian channel, therefore the rotation scheme provides no gain.

3. Constellation Rotation

This section outlines the computation of the optimal rotation angle θ_{opt}, for the interleaved system presented in the previous section. We develop a general method that can be used for any linear multilevel constellation, however for simplicity of presentation we provide only detailed analysis for the QPSK (4QAM) scheme.

The average symbol error probability can be approximated quite accurately at high signal-to-noise ratios (SNRs) by the union bound, which is given as

$$P_s \leq \frac{1}{2^q} \sum_{i=1}^{2^q} \sum_{\hat{s_i} \neq s_i} P_2(s_i \rightarrow \hat{s_i}) \tag{8}$$

where P_s is the average symbol error probability, 2^q is the number of constellation points and q is the spectral efficiency of the modulation scheme. The metric $P_2(s_i \rightarrow \hat{s_i})$ is defined as the pairwise error probability when the transmitted symbol s_i is detected as $\hat{s_i}$. For Rayleigh fading channels and a two branch orthogonal transmit diversity (two parallel Rayleigh fading channels) this metric can be approximated by [7], [8]

$$P_2(s_i \rightarrow \hat{s_i}) \leq \frac{1}{2}\left(\frac{1}{1+\gamma_s \left|x_i - \hat{x_i}\right|^2}\right)\left(\frac{1}{1+\gamma_s \left|y_i - \hat{y_i}\right|^2}\right) \tag{9}$$

where γ_s is the average SNR defined as $\gamma_s = \frac{E[h^2]E_s}{8N_0}$. We assume the expected value of fading power to be one i.e. $E[h^2] = 1$ and the average symbol energy E_s constrained to unity i.e. $E_s = 1$. The coefficients x_i and y_i represent the rotated I and Q components as given in (3).

Using (3), we can write the pairwise error probability in (9) as

$$P_2\left(s_i \rightarrow \hat{s_i}\right) \leq L_1 L_2 \tag{10}$$

where in the above equation

$$L_1 = \left(\frac{1}{1+\gamma_s \left|(a_i - \hat{a_i})\cos\theta + (b_i - \hat{b_i})\sin\theta\right|^2}\right)$$
$$L_2 = \left(\frac{1}{1+\gamma_s \left|-(a_i - \hat{a_i})\sin\theta + (b_i - \hat{b_i})\cos\theta\right|^2}\right) \tag{11}$$

For the case of no rotation i.e. $\theta = 0$, the I and Q components become

$$x_i = a_i$$
$$y_i = b_i$$

Substituting these in (9) or equivalently putting $\theta = 0$ in (11), we get

$$L_1 = \left(\frac{1}{1+\gamma_s \left|(a_i - \hat{a}_i)\right|^2} \right)$$
$$L_2 = \left(\frac{1}{1+\gamma_s \left|(b_i - \hat{b}_i)\right|^2} \right) \tag{12}$$

By comparing (11) and (12), we see that the error probability is dependent on angle θ. Thus we have to find a value of θ that minimizes the error probability or maximizes the diversity gain of the system.

It can be easily seen that for MPSK schemes, the minimization of error probability P_s is equivalent to the minimization of the following cost function

$$J = \sum_{\hat{s}_i \neq s_i} P_2(s_i \rightarrow \hat{s}_i) \tag{13}$$

Thus for MPSK any chosen value of $i = 1, 2, \cdots, 2^q$ would yield the same result (due to symmetry), however for MQAM we choose $i : \arg_{\min, \forall i} \left\{ s_i^2 \right\}$, i.e. any inner constellation point with the worst symbol error probability.

The minimization of the cost function J can be performed numerically by using the gradient search algorithm. The gradient descent algorithm can be explained as [9, p.120],[10]

$$\theta[k+1] = \theta[k] - \mu \frac{\partial J}{\partial \theta} \tag{14}$$

where it can be shown that for an arbitrarily small value of μ and a large number of iterations, θ converges to a value θ_{opt}, that minimizes J and consequently P_s. The gradient descent algorithm only uses local information i.e. an update from $\theta[k]$ to $\theta[k+1]$ depends only on $\theta[k]$ and its derivative at k. If the cost function has many minima, the gradient descent algorithm gets trapped at a minimum that is not (globally) the smallest. In that case choosing an initial value of θ is very critical.

As an example we consider the QPSK (4QAM) scheme. Using (10) and (11), the cost function in (13) can be written as

$$J = \frac{2}{1+2\gamma_s+\gamma_s^2 \sin^2 2\theta} + \frac{1}{1+4\gamma_s+4\gamma_s^2 \left(1-\sin^2 2\theta\right)} \tag{15}$$

Taking the derivative with respect to θ, we get

$$\frac{\partial J}{\partial \theta} = \frac{K_1}{\left(1+2\gamma_s+\gamma_s^2 \sin^2 2\theta\right)^2} + \frac{K_2}{\left(1+4\gamma_s+4\gamma_s^2 \left(1-\sin^2 2\theta\right)\right)^2} \tag{16}$$

where in the above equation

$$K_1 = -2\gamma_s^2 \sin(4\theta)$$
$$K_2 = 8\gamma_s^2 \sin(4\theta)$$

Using the equation (16) in (14) we get the value of $\theta_{opt}^{QPSK} = 30.3°$. By using a similar procedure we get the following values of θ_{opt}

$$\theta_{opt}^{mod} = \begin{cases} 9.5°, & \text{for mod = 8PSK} \\ 39.5°, & \text{for mod = 16QAM} \end{cases}$$

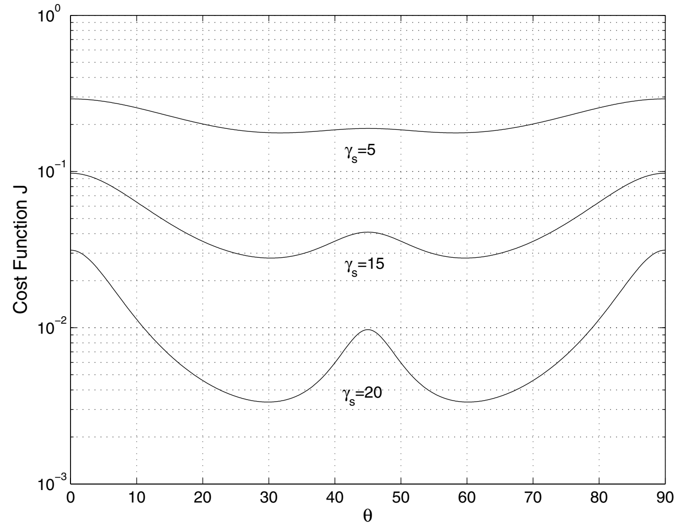

Fig. 6. Cost function J of QPSK for various rotation angles

Figure 6 gives the cost function J of QPSK as a function of rotation angle. As can be seen from Figure 6, there is only one global minima therefore any chosen intial value of θ would result in a correct solution. In this paper we use $\mu = 0.001$ and perform a gradient search over $50,000$ iterations.

4. Simulation Results

To illustrate the performance of optimum rotation we consider QPSK (4QAM), 8PSK and 16QAM modulation schemes over block Rayleigh fading channels. We assume a fully interleaved system, thus there is no inter-symbol-interference (ISI) among various transmitted symbols. The effect of interleaver depth with rotated QPSK was examined in [5].

Figure 7 gives the average symbol error probability (SEP) for the system employing QPSK modulation. The optimum rotation scheme is compared with the suboptimum scheme of [5] and is seen to provide an improvement of about 0.5 dB at high signal-to-noise (SNR) ratios.

The performance of 8PSK and 16QAM schemes with optimum rotation angles is presented in Figures 8 and 9, respectively. As compared to the conventional schemes, the optimally rotated system performs 1.37 dB better for 8PSK and 1.46 dB better for 16QAM at a SEP of 10^{-1}.

As can be seen from the above performance curves, high gains for optimal rotation are obtained at large SNR values. The actual performance improvement depends on the product distances of each individual modulation schemes and hence differs from one modulation scheme to the other. For a typical voice communication system and an average bit error rate of around 10^{-3} i.e. a SEP of around 10^{-2} [11] is required

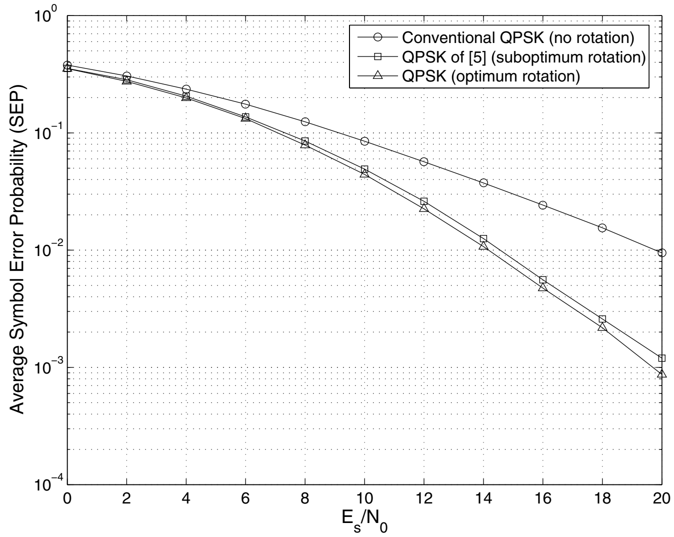

Fig. 7. Average SEP for QPSK

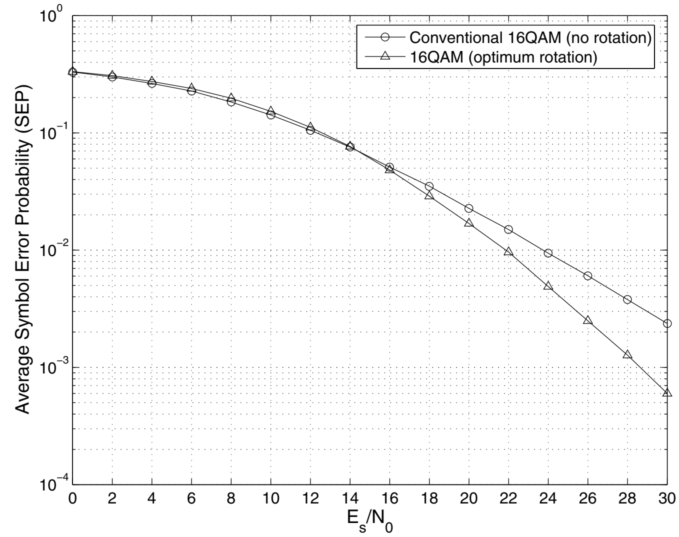

Fig. 9. Average SEP for 16QAM

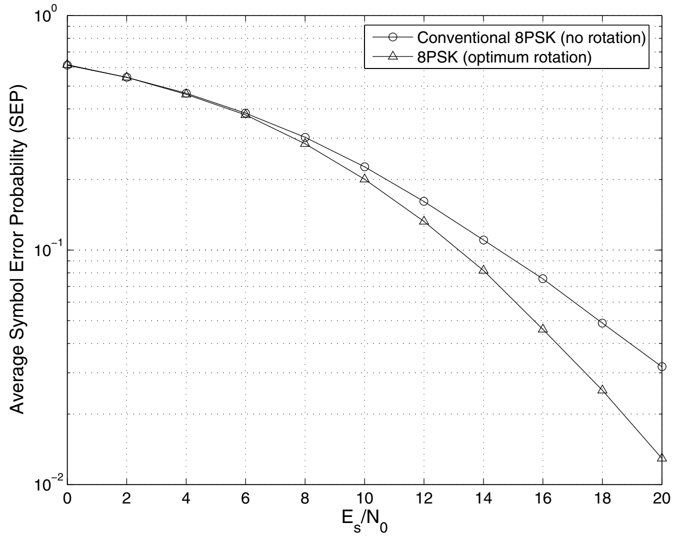

Fig. 8. Average SEP for 8PSK

for acceptable performance. Therefore, considerable power savings can be obtained by using the rotated scheme for a digital communication system with voice data traffic.

5. CONCLUSIONS

This paper presents rotated I and Q interleaving for multi-level complex linear modulation schemes, for block Rayleigh fading wireless communication channels. The optimum rotation angle was found by optimization of the upper bound on the SEP, using the gradient search algorithm. The proposed method can be used to compute the optimum rotation angle for any MPSK/MQAM interleaved scheme. The rotated and interleaved system when used with an optimum rotation provides performance gains of around 1.37 dB for 8PSK and 1.46 dB for the 16QAM at a symbol error rate of 10^{-1}.

6. ACKNOWLEDGMENTS

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