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BER Analysis of BPSK and QPSK Constellations in the Presence of ADC Quantization Noise

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***Abstract*—Quantization noise (QN) due to limited analog to digital converter (ADC) word length gives rise to degradation in system performance, especially in high frequency communication systems with sampling rates in the range of giga samples per second (Gsps). In this paper, expressions are derived for the evaluation of the bit error rate (BER) of binary phase shift keying (BPSK) and quadrature PSK (QPSK) signal constellations as a function of ADC word length (N_{ADC}) and signal to noise ratio (SNR) due to additive white Gaussian noise (AWGN). It is shown that unlike the AWGN case, system performance is dependent on the signal constellation rotation when QN due to the ADC is dominant. The newly derived expressions are then used to investigate the impact of constellation rotation on the system BER performance. It is shown that considerable performance gains can be achieved with an appropriate rotation of the signal constellation.**

I. Introduction

One of the major sources of front end power consumption in high frequency wideband communication systems, such as those operating in the ultra wideband and 60 GHz regime, is the ADC. The power consumption in an ADC is directly proportional to the product of the number of quantization levels and sampling frequency [1]. Thus with Gsps sampling, the number of bits in the ADC must be kept to a minimum. The single carrier (SC) schemes, due to their low peak-to-average power ratio (PAPR), are robust to several radio frequency (RF) circuit imperfections [2], [3] and are therefore considered in this paper.

Monte Carlo (MC) simulations are often used to evaluate the system performance in terms of average probability of error in the presence of ADC QN. A simulation based analysis for multi-carrier and SC schemes in the presence of ADC QN can be found in [3]. To the best of our knowledge, no analytical expressions for SC schemes in the presence of ADC QN have been reported in literature.

In this paper, exact expressions are derived for the pairwise error probability (PEP) in the presence of ADC QN over AWGN channels. These expressions for the PEP are then used in conjunction with the union bound (UB) for the BER analysis of BPSK and QPSK signal constellations in various fading and non-fading scenarios. For the fading analysis the Nakamgami-m distribution is used. The derived expressions can help the system designer to readily perform trade-offs in terms of N_{ADC} and SNR. Another use could be to choose the minimum N_{ADC} required to achieve a desired BER at a given SNR. It is also shown that in the presence of ADC QN a change in constellation rotation angle affects the system performance, which is quite interesting as the rotation of the signal constellation has no impact in AWGN and fading channels. The newly derived expressions are then used to investigate the impact of constellation rotation on QPSK. It is seen that the change in BER due to rotation is significant when QN is dominant.

The exact expressions for the BER of BPSK signals are presented in Section III. Section IV provides the BER analysis of the QPSK signal constellation. In Section V the impact of constellation rotation on the QPSK modulation scheme is investigated. Conclusions are drawn in Section VI.

II. Notation and System Model

The system model, notations and elementary probability relations, used repeatedly throughout the paper, are presented in this section.

A. Notation

The PDF of a random variable A denoted by $p_A(a)$ has the Fourier transform (FT) $\mathcal{F}_A(j\omega) = \int_{-\infty}^{\infty} p_A(a) e^{-j\omega a} da$, where $j^2 = -1$. Given $\mathcal{F}_A(j\omega)$, $p_A(a)$ can be obtained as $p_A(a) = \mathcal{F}^{-1}(\mathcal{F}_A(j\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{F}_A(j\omega) e^{j\omega a} d\omega$. If a random variable U is the sum of two independent random variables A and B with PDFs $p_A(a)$ and $p_B(b)$, respectively, the PDF of U is given by

$$p_U(u) = p_A(a) * p_B(b) = \mathcal{F}^{-1}(\mathcal{F}_A(j\omega)\mathcal{F}_B(j\omega)), \quad (1)$$

where $*$ denotes the convolution operation. The PDFs of random variables used within this paper along with their FTs are given in Table I. The Gaussian distribution is assumed to be zero mean with variance σ_N^2 which is denoted as $\mathcal{N}(0, \sigma_N^2)$. Similarly, the uniform distribution (between $\pm b$) is also zero mean with variance $\sigma_U^2 = b^2/3$, i.e., $\mathcal{U}(0, \sigma_U^2)$. The Nakagami-$m$ distribution with shape parameter m is considered. The shape parameter m can be varied to incorporate various fading conditions. For example, the Nakagami-m distribution is reduced to a Rayleigh PDF when $m = 1$ and is related to the Rician K factor as [4, p.23] $m = \frac{(1+K)^2}{(1+2K)}$. $\Gamma(.)$ denotes the gamma function and is defined as $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$.

The average symbol energy of a signal constellation set $\mathcal{S}_M \in \{s_1, \cdots, s_M\}$ is defined as $E_s = \frac{1}{M}\sum_{i=1}^{M} s_i^2$. The SNR

TABLE I
PDF AND FOURIER TRANSFORMS OF THE GAUSSIAN, THE UNIFORM AND THE NAKAGAMI-M DISTRIBUTION

Distribution	PDF	Fourier
Uniform	$p_U(u) = \begin{cases} \frac{1}{2b}, & -b \leq u \leq b, \\ 0, & \text{otherwise} \end{cases}$	$\frac{e^{j\omega b} - e^{-j\omega b}}{2j\omega b}$
Gaussian	$p_N(n) = \frac{e^{-\frac{x^2}{2\sigma_N^2}}}{\sqrt{2\pi\sigma_N^2}}$	$e^{-\frac{\sigma_n^2 w^2}{2}}$
Nakagami	$p_H(h) = \frac{2m^m h^{2m-1}}{\Gamma(m)} e^{-mh^2}$	-

is defined as $\gamma = \frac{E_s}{\sigma_N^2}$. In this paper, all signal constellations are assumed to be unity constrained, i.e., $E_s = 1$. Using the relation $E_s = \log_2(M) E_b$, we can write $\gamma_b = \frac{E_b}{N_0} = \frac{\gamma}{2\log_2(M)}$, where N_0 denotes the noise power spectral density (PSD) and is related to the noise variance as $N_0 = 2\sigma_N^2$. The BER for any signal constellation set $\mathcal{S}_M$, in the presence of in-phase (I) and the quadrature-phase (Q) noise components Z^I and Z^Q can be upper bounded by the well known UB as [5, p.194]

$$P_b \leq \frac{1}{M\log_2(M)} \sum_{m=1}^{M} \sum_{k=1, k\neq m}^{M} \mathcal{H}(s_m, s_k) P_e(s_m, s_k), \quad (2)$$

where $\mathcal{H}(s_m, s_k)$ denotes the Hamming distance between binary mappings associated with the symbols s_m and s_k, respectively. The PEP $P_e(s_m, s_k)$ for one-dimensional (1-D) and two-dimensional (2-D) signal constellations is given as $P_e(s_m, s_k) = \int_{\mathcal{R}(s_m, s_k)} p_{Z^I}(z^I) dz^I$ and $P_e(s_m, s_k) = \iint_{\mathcal{R}(s_m, s_k)} p_{Z^I, Z^Q}(z^I, z^Q) dz^I dz^Q$ respectively, where $\mathcal{R}(s_m, s_k)$ denotes the region in which s_m is detected as s_k and $p_{Z^I, Z^Q}(z^I, z^Q)$ is the joint PDF for the I and Q noise components. The error function erf(x) and the Q function are defined as $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\left(\frac{t^2}{2}\right)} dt$, respectively, and are related as $\text{erf}(x) = 1 - 2Q\left(\sqrt{2}x\right)$. The real and imaginary parts of any symbol s_m represent the I and Q components.

B. System Model

The equivalent baseband system model for a communication system with the assumption of coherent detection under the influence of fading H, AWGN N and ADC noise U is shown in Figure 1. The AWGN is assumed to be zero mean with a total variance of σ_N^2, i.e., if we have two branches I and Q the noise variance will be $\sigma_N^2/2$ per branch. Furthermore, it is assumed that perfect channel state information (CSI) is available at the receiver. The automatic gain control (AGC) is assumed to be working perfectly to bring the received signal within the operating range of the ADC so that there is no clipping. The additive quantization noise due to the ADC is well known to follow a uniform distribution [1, p.186]. Therefore, the random variables U^I and U^Q are uniformly distributed between $\pm b$ with noise variance $\sigma_{U^I}^2 = \sigma_{U^Q}^2 = \sigma_U^2$. The parameter b is related to the noise variance as $b = \sqrt{3 \cdot \sigma_U^2}$. The quantization noise for an ADC operating at a full scale voltage of $\pm V_s$ can be written as [1, p.186] $\sigma_U^2 = \frac{V_s^2}{3 \cdot \left(2^{2N_{ADC}}\right)}$. Assuming $V_s = 1$

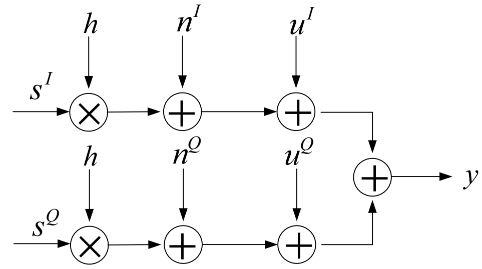

Fig. 1. Baseband equivalent system model in the presence of Fading, Gaussian noise and ADC noise

V, we get

$$b = 2^{-N_{ADC}}. \quad (3)$$

When there is no fading, i.e., for an additive noise channel, $h = 1$. For fading scenarios $E\left\{h^2\right\} = 1$, where $E\{.\}$ denotes the expectation. It is assumed that the received symbol y is passed to a maximum likelihood (ML) detector which is operating on a symbol by symbol basis.

III. BER ANALYSIS OF BPSK SIGNAL CONSTELLATION

In 1-D case the PDF of $Z^I = N^I + U^I$ needs to be evaluated. Using Table I and equation (1) we can compute the PDF of Z^I [6, p.22], which can be written as

$$p_{Z^I}\left(z^I\right) = 2^{N_{ADC}-2} \left[\text{erf}\left(\frac{z^I + b}{\sqrt{2}\sigma_N}\right) - \text{erf}\left(\frac{z^I - b}{\sqrt{2}\sigma_N}\right)\right]. \quad (4)$$

The PEP is thus given as

$$P_e(s_m, s_k) = \frac{2^{N_{ADC}}}{4} \int_{\frac{\mathcal{D}(s_m, s_k)}{2}}^{\infty} \text{erf}\left(\frac{z^I + b}{\sqrt{2}\sigma_N}\right) - \text{erf}\left(\frac{z^I - b}{\sqrt{2}\sigma_N}\right) dz^I, \quad (5)$$

where $\mathcal{D}(s_m, s_k)$ denotes the Euclidean distance between signal constellation points s_m and s_k. Solving the integral in (5) and substituting in (2) yields an expression for BER as a function of N_{ADC} and γ as given in (8). The BER of BPSK, using equation (8) can be written in closed form as given in (9), where equality in (9) follows from the exactness of the UB in case of binary signalling. Without QN, i.e., $N_{ADC} \to \infty$ the equation (9) reduces to

$$P_b^{NB} = \lim_{N_{ADC} \to \infty} P_b^{BPSK} = Q\left(\sqrt{\gamma}\right) = Q\left(\sqrt{2E_b/N_0}\right), \quad (10)$$

which is the exact expression for the error probability of BPSK in AWGN channels. A comparison of MC simulated and the analytic BER for BPSK modulation using various N_{ADC} values is shown in Figure 2.

Evaluation of the BER, when coherent detection is applied in fading conditions requires the computation of an average over the fading distribution, i.e.,

$$P_b^F = \int_0^\infty P_b^{BPSK} \, |h \;\; p_H(h) \, dh, \quad (11)$$

where $p_H(h)$ is the Nakagami-m fading distribution as given in Table I and $P_b^{BPSK}\,|h$ is given in (12). The integral in

$$P_b^{G1} \leq \frac{1}{M\log_2(M)} \sum_{m=1}^{M} \sum_{k=1,k\neq m}^{M} \mathcal{H}(s_m,s_k)\left(\frac{1}{2}-\frac{1}{4}\left[\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(\mathcal{D}(s_m,s_k)+2^{(1-N_{ADC})}\right)\right)+\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(\mathcal{D}(s_m,s_k)-2^{(1-N_{ADC})}\right)\right)\right]-\right.$$
$$\mathcal{D}(s_m,s_k)\,2^{(N_{ADC}-3)}\left[\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(\mathcal{D}(s_m,s_k)+2^{(1-N_{ADC})}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(\mathcal{D}(s_m,s_k)-2^{(1-N_{ADC})}\right)\right)\right]-$$
$$\left.\frac{2^{N_{ADC}}}{\sqrt{8\pi\gamma}}\left[e^{-\frac{\gamma}{8}\left(\mathcal{D}(s_m,s_k)+2^{(1-N_{ADC})}\right)^2}-e^{-\frac{\gamma}{8}\left(\mathcal{D}(s_m,s_k)-2^{(1-N_{ADC})}\right)^2}\right]\right). \quad (8)$$

$$P_b^{BPSK} = \left(\tfrac{1}{2}+2^{(N_{ADC}-1)}\right)Q\left(\sqrt{\gamma}\left(1+2^{(-N_{ADC})}\right)\right)+\left(\tfrac{1}{2}-2^{(N_{ADC}-1)}\right)Q\left(\sqrt{\gamma}\left(1-2^{(-N_{ADC})}\right)\right)-$$
$$\frac{2^{N_{ADC}}}{\sqrt{8\pi\gamma}}\left[e^{-\frac{\gamma}{2}\left(1+2^{(-N_{ADC})}\right)^2}-e^{-\frac{\gamma}{2}\left(1-2^{(-N_{ADC})}\right)^2}\right]. \quad (9)$$

$$P_b^{BPSK}|h = \frac{1}{2}-\frac{1}{4}\left[\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(2h+2^{(1-N_{ADC})}\right)\right)+\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(2h-2^{(1-N_{ADC})}\right)\right)\right]-$$
$$2h2^{(N_{ADC}-3)}\left[\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(2h+2^{(1-N_{ADC})}\right)\right)-\operatorname{erf}\left(\sqrt{\frac{\gamma}{8}}\left(2h-2^{(1-N_{ADC})}\right)\right)\right]-$$
$$\frac{2^{N_{ADC}}}{\sqrt{8\pi\gamma}}\left[e^{-\frac{\gamma}{8}\left(2h+2^{(1-N_{ADC})}\right)^2}-e^{-\frac{\gamma}{8}\left(2h-2^{(1-N_{ADC})}\right)^2}\right]. \quad (12)$$

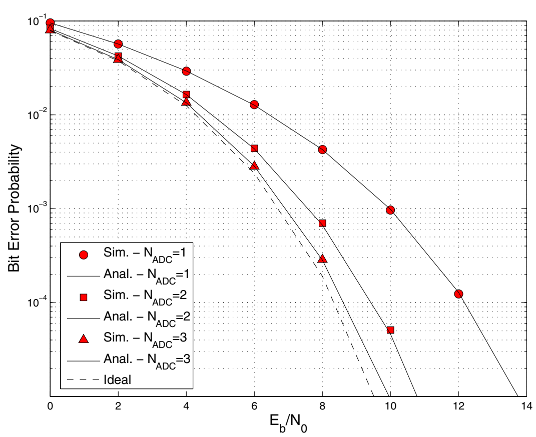

Fig. 2. Bit Error Probability for BPSK in AWGN channels for various N_{ADC} values

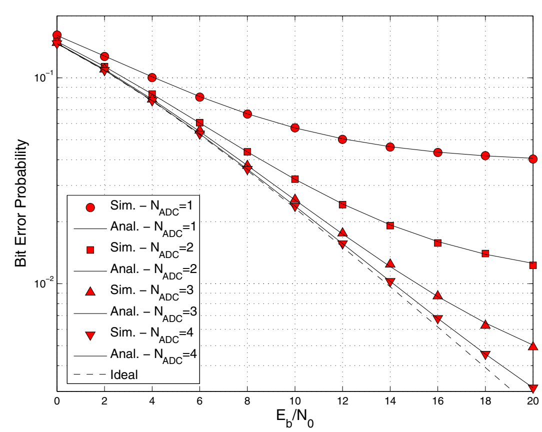

Fig. 3. Bit Error Probability for BPSK in Rayleigh fading channels for various N_{ADC} values

(11) is quite cumbersome to compute, therefore we resort to numerical integration. Thus (11) can be used to compute the BER for the BPSK modulation scheme under fading conditions. The MC simulation results and analytic BER obtained by (11) for BPSK signals when used with various ADC word lengths and over Nakagami-m fading channels with $m = 1$, i.e, Rayleigh fading is shown in Figure 3. The simulation and analytic results are seen to be in good agreement.

IV. BER Analysis of QPSK Signal Constellation

In order to find the BER of a 2-D signal constellation set, the PDF given in (4) must first be represented in terms of I and Q components. Since I and Q are independent, the combined 2-D PDF is the product of two 1-D PDFs and is given as

$$p_{Z^I,Z^Q}\left(z^I,z^Q\right) = 2^{2N_{ADC}-4}\left[\operatorname{erf}\left(\frac{z^I+2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)-\operatorname{erf}\left(\frac{z^I-2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)\right]\cdot\left[\operatorname{erf}\left(\frac{z^Q+2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)-\operatorname{erf}\left(\frac{z^Q-2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)\right].$$

Using the transformation $z^I = r\cos(\theta)$ and $z^Q = r\sin(\theta)$ and taking the Jacobian [7, p.277], the PDF can be written in

$$P_b^{QPSK} = \frac{1}{8}\sum_{m=1}^{4}\sum_{k=1,k\neq m}^{4}\mathcal{H}\left(s_m, s_k\right)\left(\int_0^{\theta_1+\frac{\pi}{2}}\int_{\left|\frac{\mathcal{D}(s_m,s_k)}{2\cos(\theta_1-\theta)}\right|}^{\infty} p_{R,\theta}\left(r,\theta\right)drd\theta + \int_0^{\frac{\pi}{2}-\theta_1}\int_{\left|\frac{\mathcal{D}(s_m,s_k)}{2\cos(\theta_1+\theta)}\right|}^{\infty} p_{R,\theta}\left(r,\theta\right)drd\theta\right). \tag{14}$$

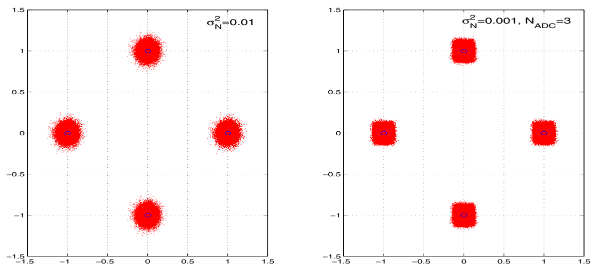

Fig. 4. Impact of ADC noise and AWGN on QPSK

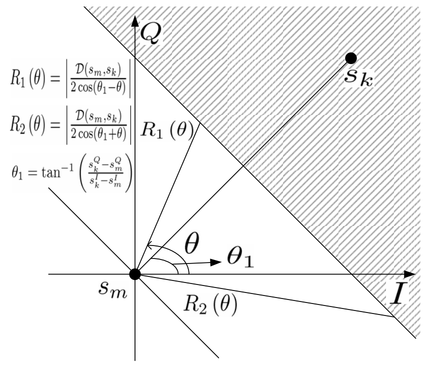

Fig. 5. Error region for PEP of 2-dimensional signal constellations in ADC noise

terms of the polar coordinates as

$$\begin{aligned} p_{R,\theta}\left(r,\theta\right) = \; & 2^{2N_{ADC}-4}r\left[\mathrm{erf}\left(\frac{r\cos(\theta)+2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)-\right. \\ & \left.\mathrm{erf}\left(\frac{r\cos(\theta)-2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)\right]\cdot\left[\mathrm{erf}\left(\frac{r\sin(\theta)+2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)-\right. \\ & \left.\mathrm{erf}\left(\frac{r\sin(\theta)-2^{-N_{ADC}}}{\sqrt{2}\sigma_N}\right)\right] \end{aligned} \tag{13}$$

In the presence of ADC QN the PDF is no longer independent of θ as in the case of AWGN which has a circular PDF $p_{R,\theta}\left(r,\theta\right) = \frac{r}{2\pi\sigma_N^2}e^{-\frac{r^2}{2\sigma_N^2}}$. This can be explained intuitively with the help of Figure 4, in which, a QPSK signal constellation perturbed by Gaussian noise (left) and ADC plus Gaussian noise (right) is shown. It is clear from the Figure that the Gaussian noise is uniformly distributed and therefore signal constellation rotation will have no impact on the system performance. This however, is not the case when ADC noise is also present. The noise distribution in the presence of ADC QN takes on a rectangular shape and thus a rotation of the signal constellation will affect the BER. The PDF given in (13) is however symmetrical about the x- and y-axis as can also be seen from Figure 4. Thus to evaluate the PEP for symbol s_m and s_k we should integrate over the shaded region depicted in Figure 5. In order to simplify the evaluation, coordinates of s_m are taken as the origin. After some straight forward coordinate geometry we obtain $R_1\left(\theta\right) = \left|\frac{\mathcal{D}(s_m,s_k)}{2\cos(\theta_1-\theta)}\right|$, $R_2\left(\theta\right) = \left|\frac{\mathcal{D}(s_m,s_k)}{2\cos(\theta_1+\theta)}\right|$ and $\theta_1 = \tan^{-1}\left(\frac{s_k^Q-s_m^Q}{s_k^I-s_m^I}\right)$. Note that $R_2\left(\theta\right)$ follows from the noise symmetry around the x-axis, $|.|$ denotes the absolute value and is taken to prevent $R_1\left(\theta\right)$ and $R_2\left(\theta\right)$ from becoming negative. Using this formulation the BER of QPSK signal constellation is given by (14). The integral in (14) is quite difficult to evaluate analytically but can be efficiently computed using standard numerical integration techniques. Figure 6 gives a comparison of the analytical results obtained with (14) and the MC simulation results as a function of various values of N_{ADC} and E_b/N_0 for Gray mapped QPSK signal constellation. It can be seen that upper

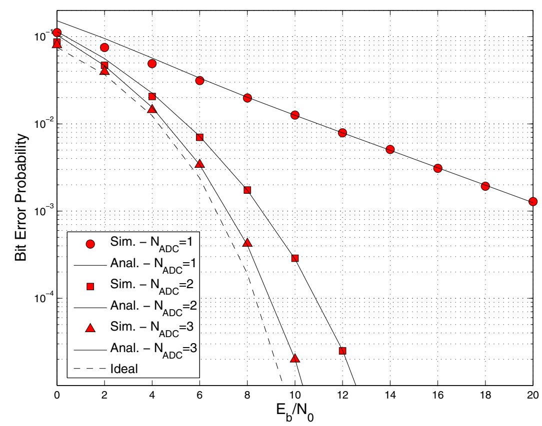

Fig. 6. Bit Error Probability for Gray coded QPSK in AWGN channels for various N_{ADC} values

bounds are quite tight at higher signal to noise ratios and practical BERs of interest, i.e., less than 10^{-2}.

V. Constellation Rotation

The sum of Gaussian and ADC QN has a PDF which exhibits non-uniform angular distribution. Hence, the BER performance is no longer independent of the signal constellation rotation before the ADC. To investigate the impact of signal constellation rotation we extend the PEP given in (14) to incorporate the constellation rotation θ_R. For clockwise rotation over an angle θ_R, the mapped symbols s_m in an MPSK signal constellation are taken from the set $\mathcal{S}_M^{\theta_R} \in \left\{s_k = e^{j2\pi(k-1)/M-j\theta_R} : k = 1, 2, \cdots, M\right\}$. The rotated and un-rotated I and Q components are related as

$$\begin{aligned} s_m^{IR} &= s_m^I\cos\left(\theta_R\right) + s_m^Q\sin\left(\theta_R\right) \\ s_m^{QR} &= -s_m^I\sin\left(\theta_R\right) - s_m^Q\cos\left(\theta_R\right) \end{aligned} \tag{15}$$

$$P_b^{QPSK,\theta_R} = \frac{1}{8}\sum_{m=1}^{4}\sum_{k=1,k\neq m}^{4}\mathcal{H}\left(s_m,s_k\right)\left(\int_0^{\theta_1^R+\frac{\pi}{2}}\int_{\left|\frac{\mathcal{D}(s_m,s_k)}{2\cos\left(\theta_1^R-\theta\right)}\right|}^{\infty} p_{R,\theta}\left(r,\theta\right)dr\,d\theta + \int_0^{\frac{\pi}{2}-\theta_1^R}\int_{\left|\frac{\mathcal{D}(s_m,s_k)}{2\cos\left(\theta_1^R+\theta\right)}\right|}^{\infty} p_{R,\theta}\left(r,\theta\right)dr\,d\theta\right). \quad (16)$$

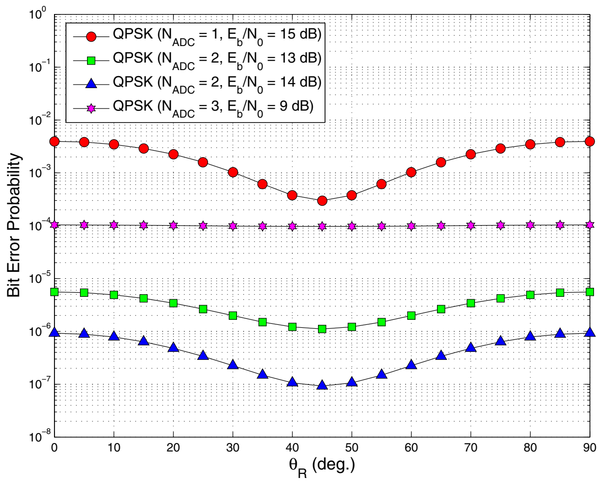

Fig. 7. Change in the BER with different rotation angles for QPSK with various N_{ADC}

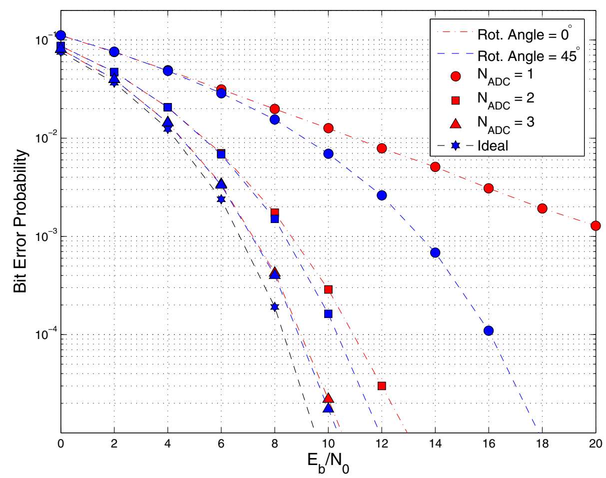

Fig. 8. Rotation gain for QPSK signal constellation in AWGN channel

Signal constellation rotation does not affect the Euclidean distance between symbols hence $\mathcal{D}\left(s_m,s_k\right)$ is unchanged, however, the angle θ_1 in (14) is changed. For the rotated signal constellations the BER of (14) is given as (16), where $\theta_1^R = \tan^{-1}\left(\frac{s_k^{QR}-s_m^{QR}}{s_k^{IR}-s_m^{IR}}\right)$. Thus, equation (16) can be used to investigate the impact of rotation for a QPSK signal constellations. Figure 7 gives a plot of the BER against various rotation angles of QPSK signal constellation for different values of N_{ADC}. It can be seen that when the ratio between the ADC quantization noise and the Gaussian noise is high, i.e., the noise is predominately uniform, then a rotation of the signal constellation has significant impact on the system performance. As the number of bits in the ADC are increased the combined noise term assumes a Gaussian distribution and the change in system BER is negligible. This can also be seen from Figure 8, in which the rotation gain of QPSK for various N_{ADC} values is presented. For a 1 bit ADC, the rotation can result in an improvement of about 6 dB at a BER of 10^{-3}. This however is still less than increasing the number of ADC bit from 1 to 2 in which case the improvement is 12 dB. Furthermore, the gain due to rotation for higher number of ADC bits will be only significant at very high SNRs. The rotation thus offers another parameter that can be used to tweak the BER performance in ADC noise limited scenarios. For example if the system SNR is 14 dB and a BER of 10^{-3} is desired then, one could in principal achieve this with a proper constellation rotation instead of increasing N_{ADC}.

VI. Conclusions

In this paper, the BER analysis of BPSK and QPSK modulation schemes in the presence of ADC QN was carried out for fading and non-fading scenarios. Analytical expressions for the BER in the presence of Gaussian noise and ADC QN were derived. It was shown that signal constellation rotation affects the system performance and the BER expressions were extended to incorporate the effect of signal constellation rotation. It was shown that for ADC QN limited systems an appropriate rotation of the signal constellation can yield significant performance improvements.

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